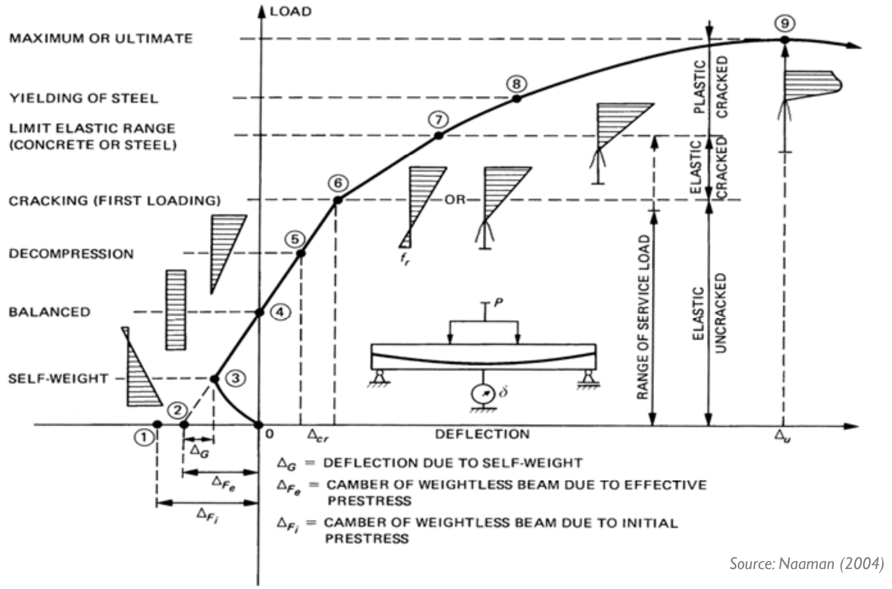
Part II: Ultimate Strength Design

Concrete and Prestressing Steel Stresses
Cracking Moment
Failure Types
Analysis for M_n – Rectangular Section
T-Section
Analysis for M_n – T- Section

Load – Deflection – Concrete Stress



Source: Naaman (2004)

Figure 5.1 Typical load-deflection curve of a prestressed concrete beam (underreinforced, with bonded tendons, first loading).

Load - Deflection

- 1 & 2: Theoretical camber (upward deflection) of prestressed beam
- 3: Self weight + Prestressing force
- 4: Zero deflection point (Balanced point) with uniform stress across section
- 5: Decompression point where tension is zero at the bottom fiber
- 6: Cracking point where cracking moment is reached
- 7: End of elastic range (the service load will not exceed this)
- 8: Yielding of prestressing steel
- 9: Ultimate strength by crushing of concrete (strain equals to 0.003)

Prestressing Steel Stress

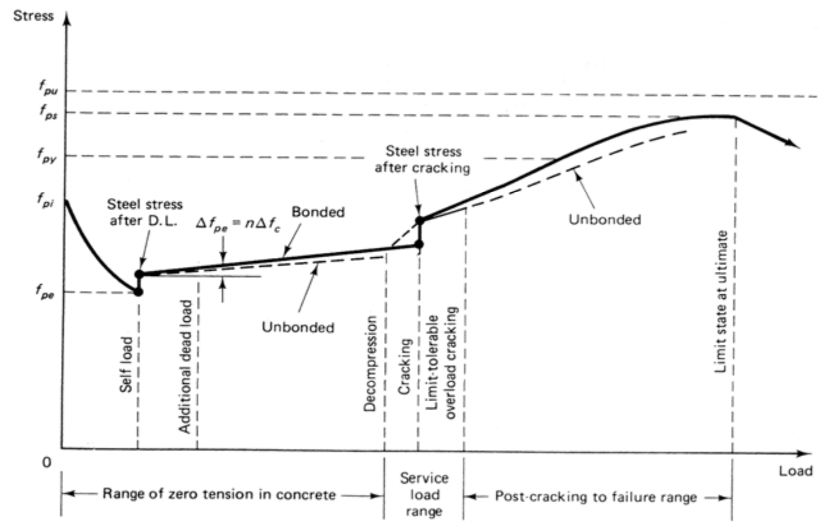


Figure 4.40 Prestressing steel stress at various load levels.

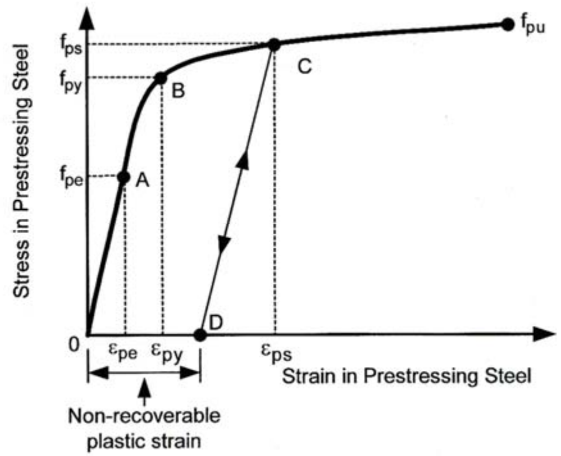
Source: Nawy (2000)

Prestressing Steel Stress

- The prestressing steel stress increases as the load increases
- Cracking of beam causes a jump in stress as additional tension force is transferred from concrete (now cracked) to prestressing steel
- At ultimate of prestressed concrete beam, the stress in steel is somewhere between yield strength f_{py} and ultimate strength f_{pu}
- Stress is lower for unbonded tendon because stress is distributed throughout the length of the beam instead of just one section as in the case of bonded tendon
- At ultimate, the effect of prestressing is lost and the section behaves just like an RC beam

Prestress Force at Ultimate

- Prestress force disappears as the prestressing steel goes into the inelastic range
- Section behaves as an RC section having prestress steel as reinforcement



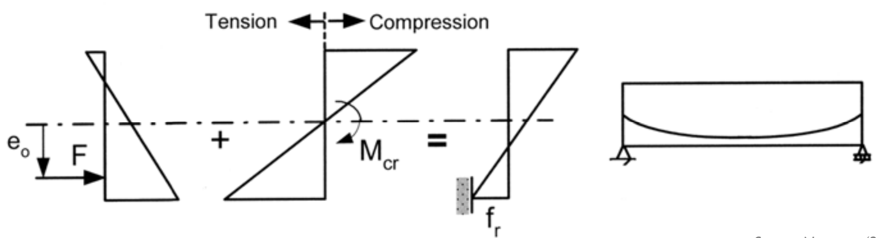
Source: Naaman (2004)

Cracking Moment

Cracking Moment

- Concrete cracks when bottom fiber reaches the tensile capacity (modulus of rupture)

$$f_r = -0.63\,(f'_c)^{0.5} \text{ MPa (5.4.2.6)}$$



Source: Naaman (2004)

Cracking Moment

- The moment at this stage is called "cracking moment" which depends on the geometry of the section and prestressing force

$$\sigma_b = \frac{F}{A_c} + \frac{Fe_o}{Z_b} - \frac{M_{cr}}{Z_b} = \frac{F}{A_c}\left(1 - \frac{e_o}{k_t}\right) - \frac{M_{cr}}{Z_b} = f_r$$

- Solve the above equation to get M_{cr}

$$M_{cr} = F(e_o - k_t) - f_r Z_b$$

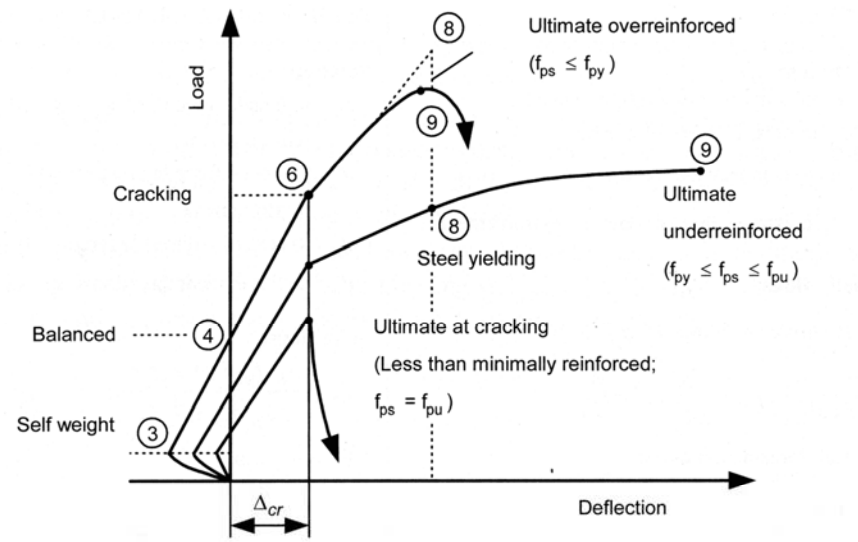
Note: Need to input f_r and k_t as negative values !!!

Ultimate Moment Capacity of T-Section

Failure Types

- This is similar to RC
- Fracture of steel after concrete cracking.
  - This is a sudden failure and occurred because the beam has too little reinforcement
- Crushing of concrete after some yielding of steel.
  - This is called tension-controlled.
- Crushing of concrete before yielding of steel.
  - This is a brittle failure due to too much reinforcement. It is called overreinforced or compression-controlled.

Failure Types



Source: Naaman (2004)

Figure 5.5 Typical change in load-deflection curve with an increase in the amount of reinforcement.

Analysis for Ultimate Moment Capacity

- Design Criteria

$$\varnothing M_n > M_u$$

Resistance Factor

Nominal Moment Capacity

Moment due to Factored Load

- Analysis assumptions
  - Plane section remains plane after bending (linear strain distribution)
  - Perfect bond between steel and concrete (strain compatibility)
  - Concrete fails when the strain is equal to 0.003
  - Tensile strength of concrete is neglected at ultimate
  - Use rectangular stress block to approximate concrete stress distribution in the compression zone

Analysis for Ultimate Moment Capacity

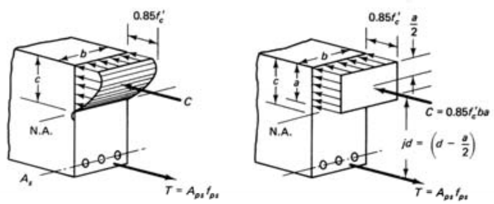
- Recall from RC Design that the followings must be satisfy <u>at all times,</u> no matter what happens:
  - EQUILIBRIUM OF FORCES
  - STRAIN COMPATIBILITY
- They also hold in Prestressed Concrete!

Equilibrium - *Forces*

- For equilibrium, there are commonly 4 forces
  - Compression in concrete
  - Compression in Nonprestressed reinforcement
  - Tension in Nonprestressed reinforcement
  - Tension in Prestressed reinforcement
- For concrete compression, we still use the ACI's rectangular stress block

Equilibrium – *Concrete Forces*

- For concrete compression, we still use the ACI's rectangular stress block



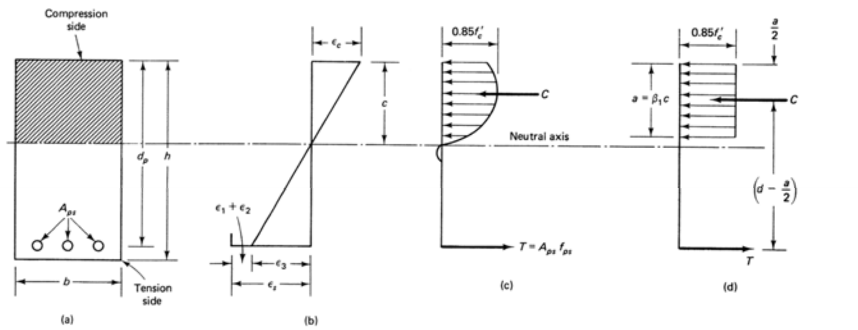


Figure 4.44 Stress and strain distribution across beam depth. (a) Beam cross section. (b) Strains. (c) Actual stress block. (d) Assumed equivalent stress block.

Source: Nawy (2000).

Equilibrium – *Concrete Forces*

$$\beta_1 = \begin{cases} 0.85 & f'_c \le 28 \text{ MPa} \\ 0.85 - 0.05\left(\dfrac{f'_c - 28}{7}\right) & 28 \le f'_c \le 56 \text{ MPa} \\ 0.65 & f'_c \ge 56 \text{ MPa} \end{cases}$$

β_1 equals to 0.85 for f'_c < 28 MPa

It decreases 0.05 for every 7 MPa increase in f'_c

until it reaches 0.65 at f'_c > 56 MPa

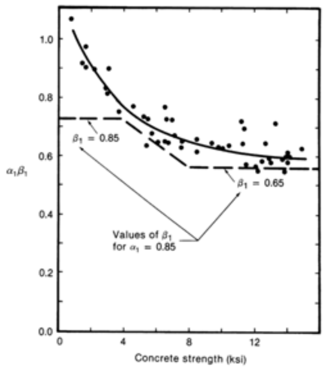


Fig. 4–13
Values of $\alpha_1\beta_1$ from tests of concrete prisms.

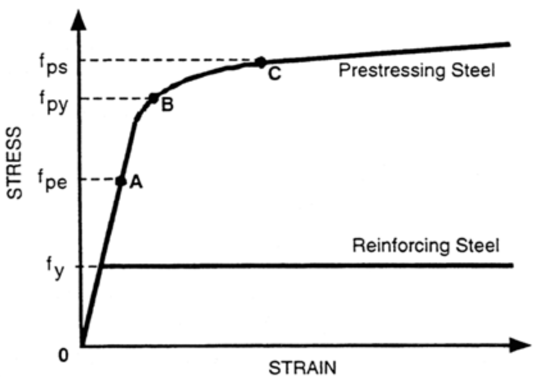
Source: MacGregor and Wight (2005).

Equilibrium – *Nonprestressed Steel Forces*

- For tension and compression in nonprestressed reinforcement, we follow the same procedure as in RC design:
  - Assume that the steel yield first; i.e.
    $T_s = A_s f_y$ or $C_s = A_s' f_y'$
  - Check the strain in reinforcement to see if they actually yield or not, if not, calculate the stress based on the strain at that level & revise the analysis
    to find new value of neutral axis depth, c
    $T_s = A_s f_s = A_s E_s \varepsilon_s = A_s E_s \cdot 0.003(c-d)/c$

Equilibrium – *Prestressed Steel Forces*

- For tension in prestressing steel, we observe that we <u>cannot</u> assume the behavior of prestressing steel (which is high strength steel) to be elastic-perfectly plastic as in the case of steel reinforcement in RC



Source: Naaman (2004)

Equilibrium – *Prestressed Steel Forces*

- At ultimate of prestressed concrete beam, the stress in steel is clearly not the yield strength but somewhere between yield strength f_{py} and ultimate strength f_{pu}
- We called it f_{ps}
- The true value of stress is difficult to calculate (generally requires *nonlinear moment-curvature analysis*) so we generally estimate it using semi-empirical formula
  - ACI → Bonded Tendon or Unbonded Tendon
  - AASHTO → Bonded Tendon or Unbonded Tendon

Ultimate Stress in Steel: f_{ps}

- AASHTO LRFD Specifications
- For Bonded tendon only (5.7.3.1.1) and for $f_{pe} > 0.5f_{pu}$

$$f_{ps} = f_{pu}\left(1 - k\frac{c}{d_p}\right)$$

$$k = 2\left(1.04 - \frac{f_{py}}{f_{pu}}\right)$$

Table C5.7.3.1.1-1 - Values of k

Type of Tendon	f_{py}/f_{pu}	Value of k
Low relaxation strand	0.90	0.28
Stress-relieved strand and Type 1 high-strength bar	0.85	0.38
Type 2 high-strength bar	0.80	0.48

Source: AASHTO (2000)

- Note: for preliminary design, we may conservatively assume $f_{ps}=f_{py}$ (5.7.3.3.1)
- For Unbonded tendon, see 5.7.3.1.2

Strain Compatibility

- Notes on Strain Compatibility

- The strain in top of concrete at ultimate is 0.003
- We can use *similar triangle* to find the strains in concrete or reinforcing steel at any levels from the top strain
- We need to add the tensile strain due to prestressing (occurred before casting of concrete in pretensioned or before grouting in posttensioned) to the strain in concrete at that level to get the true strain of the prestressing steel

Maximum & Minimum Reinforcement

- Maximum Reinforcement (5.7.3.3.1)
  - The maximum of nonprestressed and prestressed reinforcement shall be such that $c/d_e \leq 0.42$ to prevent compression failures
  - c/d_e = ratio between neutral axis depth (c) and the centroid depth of the tensile force (d_e)

- Minimum Reinforcement (5.7.3.3.2)
  - The minimum of nonprestressed and prestressed reinforcement shall be such that
    - $ØM_n > 1.2M_{cr}$ (M_{cr} = cracking moment), or
    - $ØM_n > 1.33M_u$ (M_u from Strength Load Combinations)
  - This is to prevent abrupt failure immediately after cracking

Resistance Factor ϕ

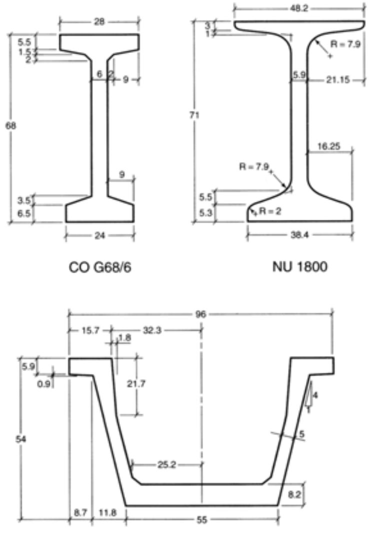
Section Type	Resistance Factor, Ø		
	RC and PPC w/ *PPR* < 0.5	PPC with 0.5< *PPR* < 1	PC (*PPR* = 1.0)
Under-Reinforced Section $c/d_e \leq 0.42$	0.90	0.90	1.00
Over-Reinforced Section $c/d_e > 0.42$	Not Permitted	0.70	0.70

- Note: if $c/d_e > 0.42$ the member is now considered a compression member and different resistance factor applies (see 5.5.4.2)
- AASHTO does not permit the use of over-reinforced RC (defined as sections with *PPR* < 0.5) sections

Ultimate Moment Capacity of T-Section

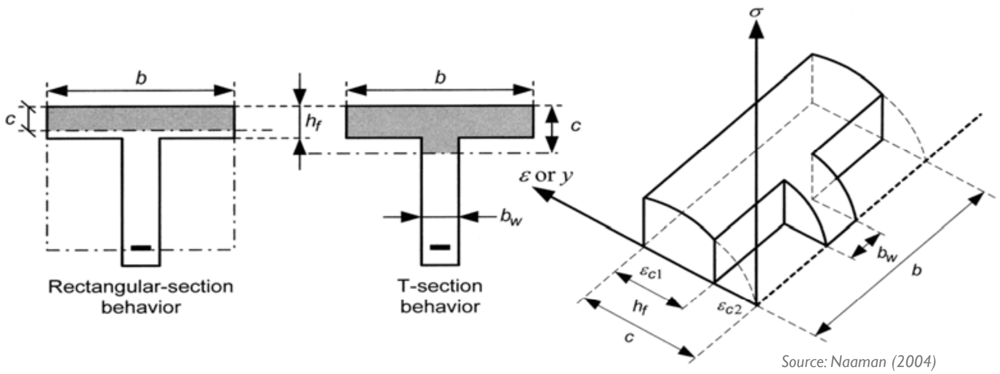
Rectangular vs. T-Section

- Most prestressed concrete beams are either I-Shaped or T-shaped (rarely rectangular) so they have larger compression flange
- If the neutral axis is in the flange, we called it rectangular section behavior. But if the neutral axis is below the flange of the section, we call it T-section behavior
- This has nothing to do with the overall shape of the section !!!



Source: Nawy (2000)

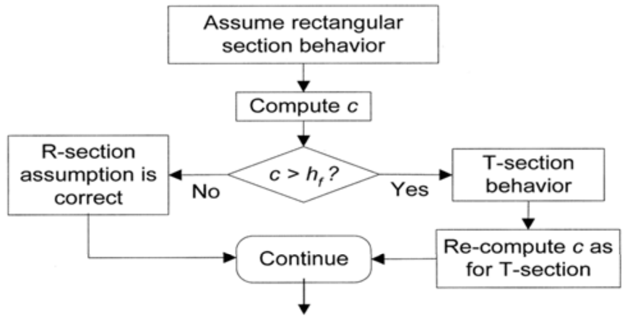
Rectangular vs. T-Section



Source: Naaman (2004)

- If it is a T-Section behavior, there are now two value of widths, namely b (for the top flange), and b_w (web width)
- We need to consider nonuniform width of rectangular stress block

Rectangular vs. T-Section



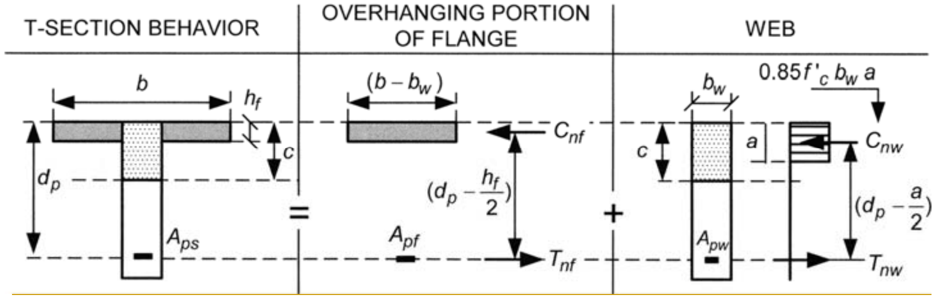
Source: Naaman (2004)

Figure 5.20 Steps to follow prior to considering T-section behavior.

- We generally assume that the section is rectangular first and check if the neutral axis depth (c) is above or below the flange thickness, h_f
- Note: ACI method checks $a=\beta_1 c$ with h_f, which may give slightly different result when $a < h_f$ but $c > h_f$

T-Section Analysis

- We divide the compression side into 2 parts
  - Overhanging portion of flange (width = b-b_w)
  - Web part (width = b_w)



Source: Naaman (2004)

T-Section Analysis

From equilibrium

$$0.85f'_c\, b_w \beta_1 c + 0.85f'_c (b - b_w)\beta_1 h_f = A_{ps} f_{ps} + A_s f_y - A_s' f_y'$$

For preliminary analysis, or first iteration, we may assume f_{ps} = f_{py} and solve for c

$$c = \frac{A_{ps} f_y + A_s f_y - A_s' f_y' - 0.85f'_c (b - b_w)\beta_1 h_f}{0.85f'_c\, b_w \beta_1}$$

T-Section Analysis

- For a more detailed approach, we recall the equilibrium

$$0.85f'_c\, b_w \beta_1 c + 0.85f'_c (b - b_w)\beta_1 h_f = A_{ps} f_{ps} + A_s f_y - A_s' f_y'$$

Substitute $f_{ps} = f_{pu}\left(1 - k\dfrac{c}{d_p}\right)$, Rearrange and solve for c

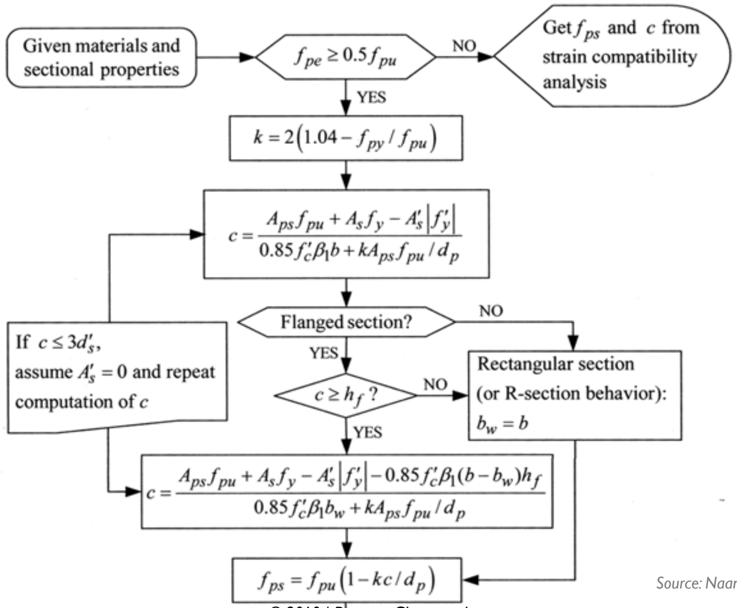
$$c = \frac{A_{ps} f_{pu} + A_s f_y - A_s' f_y' - 0.85f'_c (b - b_w)\beta_1 h_f}{0.85f'_c\, b_w \beta_1 + kA_{ps} f_{pu} / d_p}$$

T-Section Analysis

- Moment Capacity (about a/2)

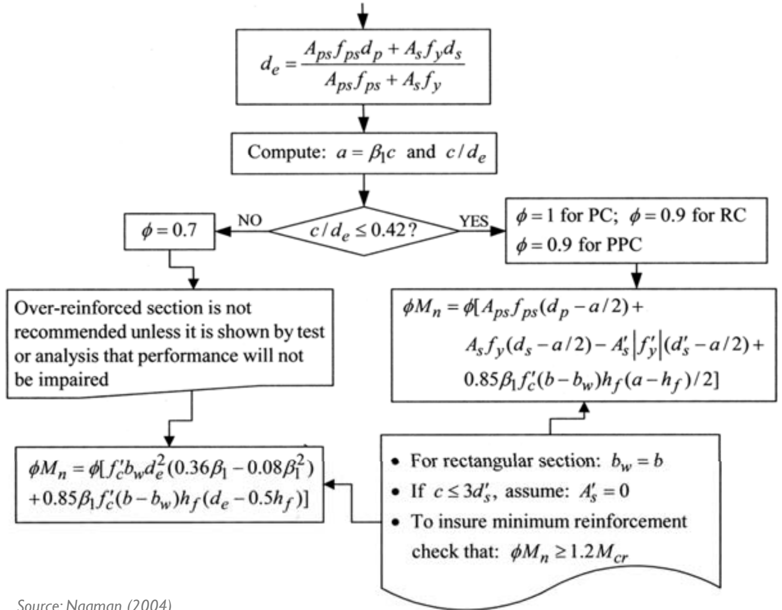
$$M_n = A_{ps} f_{ps}\left(d_p - \frac{a}{2}\right) + A_s f_y\left(d_s - \frac{a}{2}\right) - A_s' f_y'\left(d_s' - \frac{a}{2}\right)$$
$$+ 0.85f'_c (b - b_w)\beta_1 h_f\left(\frac{a}{2} - \frac{h_f}{2}\right)$$

T-Section Analysis Flowchart - *AASHTO*



Source: Naaman (2004)

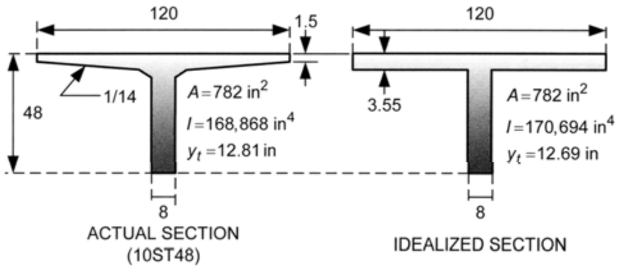
T-Section Analysis Flowchart



Source: Naaman (2004)

T-Section

- In an actual member, the section is rarely a perfect T or I shape - there are some tapering flanges and fillets. Therefore, we need to idealize the true section to simplify the analysis. Little accuracy may be lost.



Typical example of actual and idealized T section.

Source: Naaman (2004)

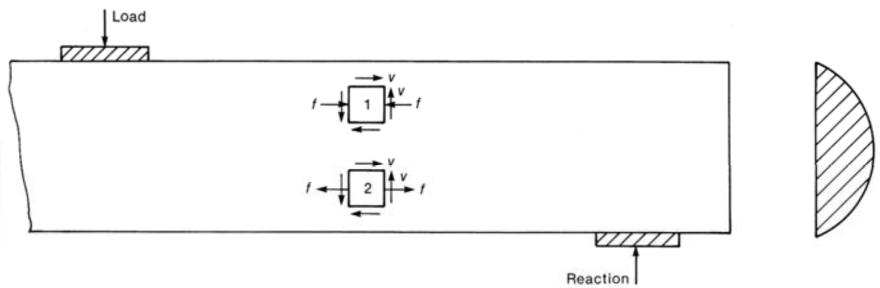
- Note that we need this for ultimate strength analysis only. We should use the true section property for the allowable stress analyses/ designs.

Part III: Shear

Principal Stresses
Shear Design Methods
ACI Traditional Design Method
AASHTO Modified Compression Field Theory

Principal Stresses in Beams

- Recall from Strength of Materials that there are two types of stress in the beam
  - Flexural stress σ = Mc/I
  - Shear Stress τ = VQ/Ib



(a) Flexural and shear stresses acting on elements in the shear span.

(b) Distribution of shear stresses.

Shear

- Elastic shear stress distribution for various sections

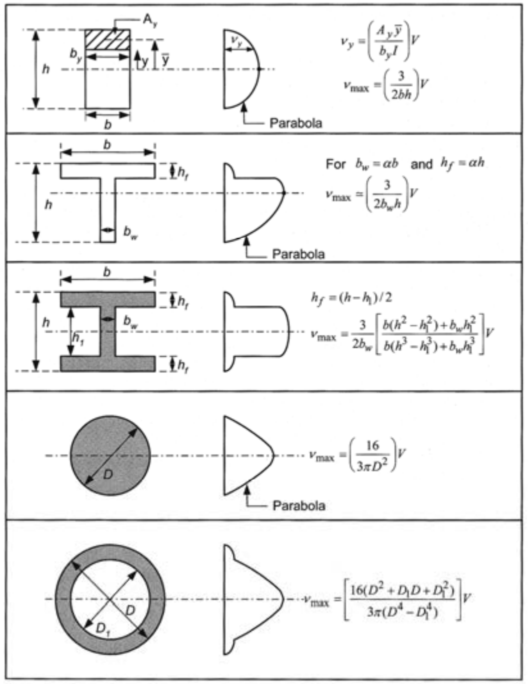
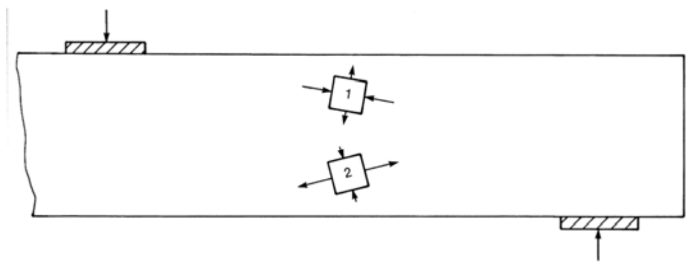


Figure 6.5 Typical shear stress diagrams and maximum shear stresses for various sections.

Principal Stresses in Beams

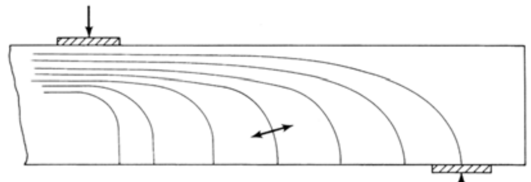
- The combination of flexural and shear stresses creates principal stresses in some incline plane
  - Principal Compressive Stress
  - Principal Tensile Stress
- Principal Compressive and Tensile stresses are *always* perpendicular to each other



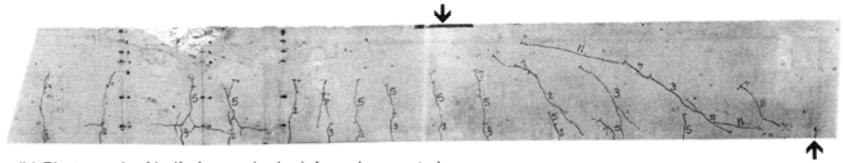
(c) Principal stresses on elements in shear span.

Principal Stresses in Beams

- Since concrete has very low tensile strength, the concrete may cracked under principal tensile strength



(a) Principal compressive stress trajectories in an uncracked beam.



(b) Photograph of half of a cracked reinforced concrete beam.

Principal Stresses in Beams

- In prestressed concrete, there is an axial force from prestressing acting on the section as well
- Effect of axial force is to reduce the principal tensile stress, thus, increasing the shear resistance

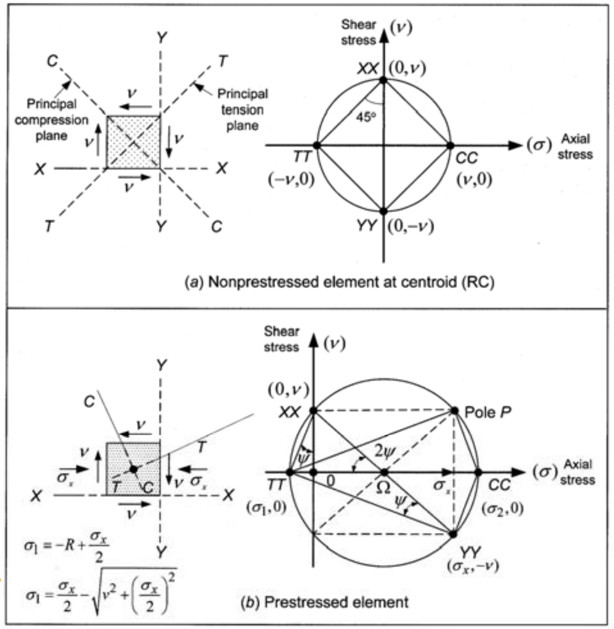
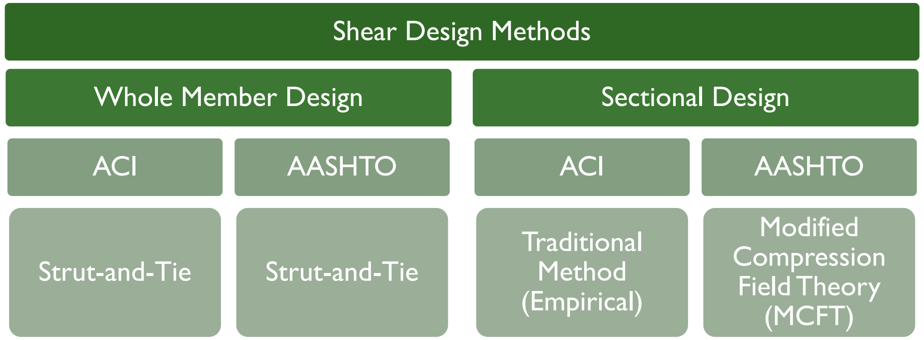


Figure 6.3 Mohr's circle for an element taken along the neutral axis.

Shear Design Methods

- Shear can be designed using whole member design method, such as STM, or designed for each section.



Design Criteria

$$V_u \le \varphi V_n$$

Nominal Shear Capacity

Ultimate Shear Force
For basic gravity case:

ACI 318-99
$1.4(V_{DL}+V_{SDL})+1.7V_{LL}$

ACI 318-02 and newer
$1.2(V_{DL}+V_{SDL})+1.6V_{LL}$

AASHTO LRFD
$1.25(V_{DC})+1.5(V_{DW})+1.75V_{LL+IM}$

Strength Reduction Factor/ Resistance Factor
For shear:

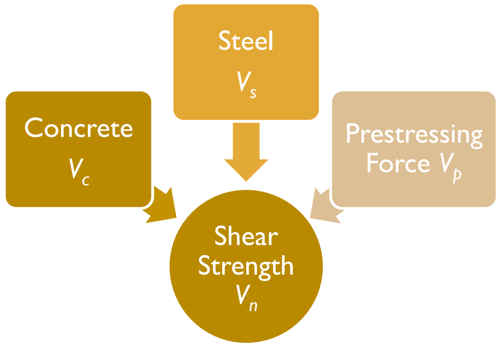
ACI 318-99
Ø=0.85

ACI 318-02 and newer
Ø=0.75

AASHTO LRFD
Ø=0.90

Components of Shear Strength

- In sectional design methods, the shear capacity of a section comes from concrete, steel stirrups, and vertical component of prestressing force
- The most difficult part to determine is V_c



Shear - MCFT

- The shear resisting mechanism in concrete is actually very complex and we do not clearly understand how to predict it
- AASHTO LRFD (5.8.3) uses new theory, called "modified compression field theory (MCFT)"
- This is based on the experimental results of concrete panel resisting pure shear and tension
- The actual theory has been simplified for use in the code
- This theory is a unified theory for concrete structures, applicable to both PC and RC

Shear - MCFT

- In AASHTO's method, the nominal shear resistance is the sum of shear strength of concrete, steel (stirrups), and shear force due to prestressing (vertical component)

$$V_n = V_c + V_s + V_p \le 0.25 f'_c b_v d_v + V_p$$

Positive when resisting the applied shear

$$V_s = \frac{A_v f_y d_v \cot\theta}{s}$$

Strut angle

$$V_c = 0.083\beta\sqrt{f'_c}\, b_v d_v$$

Factor indicating the ability of diagonally cracked concrete to transmit tension

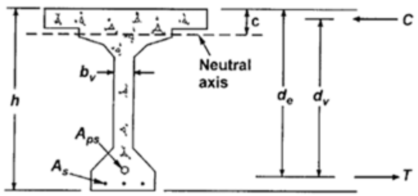


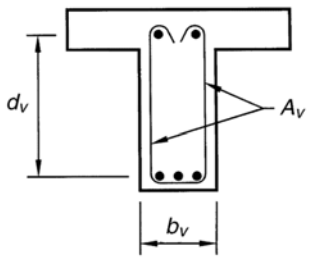
Figure C5.8.2.9-1 - Illustration of the Terms b_v and d_v.

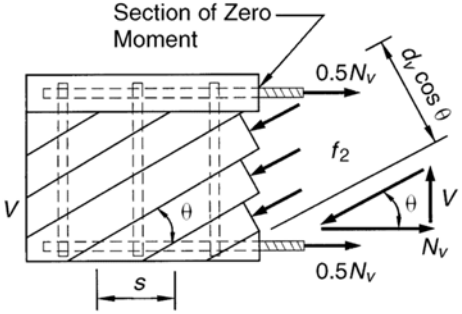
Source: AASHTO (2005)

Shear - MCFT

Effective Shear Depth

$$d_v = \max\begin{cases} d_e - a/2 \\ 0.9 d_e \\ 0.72 h \end{cases}$$





Shear - MCFT

- The value β and θ are determined for the following cases
  - For nonprestressed sections with no axial tension and having minimum transverse reinforcement or having depth less than 400 mm
    - β = 2.0
    - θ = 45°
  - For general cases, including prestressed sections, two cases must be considered
    - Section containing at least minimum transverse reinforcement
    - Section containing less than minimum transverse reinforcement
- Tables are provided for β and θ values as factors of:
  - Average longitudinal strain in concrete, ε_x
  - Ratio of v_u/f'_c
  - Crack spacing parameter, S_{xe}

Shear

- If the section has *at least* minimum transverse reinforcement

$M_u \geq V_u d_v$ Axial force, positive if tension

$\approx 0.7 f_{pu}$

$$\varepsilon_x = \frac{\left[\dfrac{|M_u|}{d_v} + 0.5N_u + 0.5|V_u - V_p|\cot\theta - A_{ps}f_{po}\right]}{2(E_s A_s + E_{ps}A_{ps})} \leq 0.001$$

Use only areas in the flexural tension side

Shear

- If the section has *less than* minimum transverse reinforcement

$M_u \geq V_u d_v$ Axial force, positive if tension

$\approx 0.7 f_{pu}$

$$\varepsilon_x = \frac{\left[\dfrac{|M_u|}{d_v} + 0.5N_u + 0.5|V_u - V_p|\cot\theta - A_{ps}f_{po}\right]}{(E_s A_s + E_{ps}A_{ps})} \leq 0.002$$

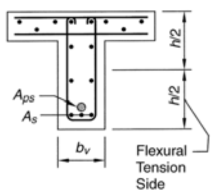
Use only areas in the flexural tension side

Shear

- If the value calculate from the previous equations was negative, need to recalculate as:

$$\varepsilon_x = \frac{\left[\dfrac{|M_u|}{d_v} + 0.5N_u + 0.5|V_u - V_p|\cot\theta - A_{ps}f_{po}\right]}{2(E_c A_{cf} + E_s A_s + E_{ps}A_{ps})} \leq 0.002$$

Area of section only on the flexural tension side (tension flange or the lower half of the section)



Shear

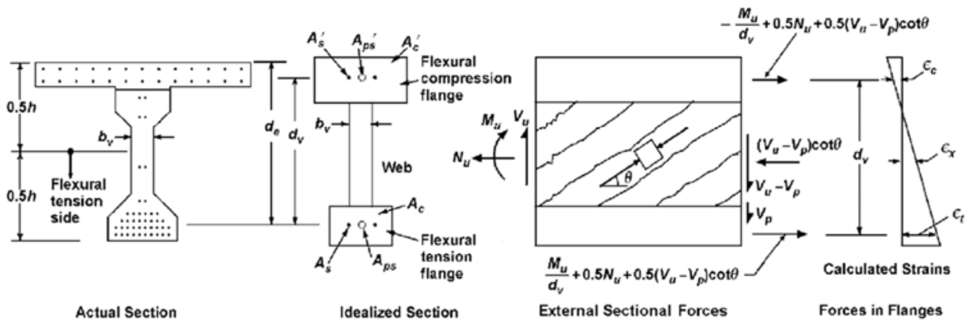


Figure C5.8.3.4.2-3 More Accurate Calculation Procedure for Determining ε_x.

Source: AASHTO (2005)

Shear

Minimum of d_v or spacing between longitudinal crack control reinforcement

$$S_{xe} = S_x \frac{35}{a_g + 16} \le 2000 \text{ mm}$$

Maximum aggregate size (mm)

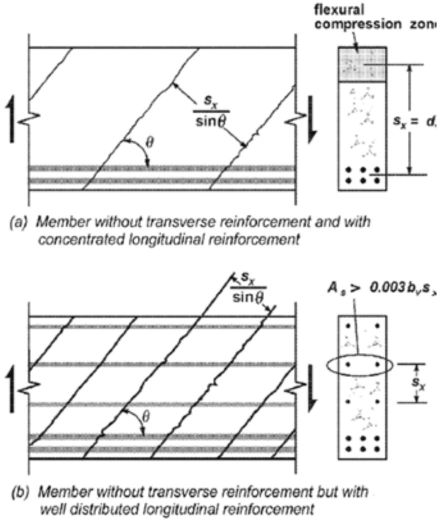


Figure 5.8.3.4.2-3 Definition of Crack Spacing Parameter s_x.

Source: AASHTO (2005)

Shear

Table 5.8.3.4.2-1 Values of θ and β for Sections with Transverse Reinforcement.

$\frac{v_u}{f'_c}$	$\varepsilon_x \times 1{,}000$								
	≤−0.20	≤−0.10	≤−0.05	≤0	≤0.125	≤0.25	≤0.50	≤0.75	≤1.00
≤0.075	22.3 6.32	20.4 4.75	21.0 4.10	21.8 3.75	24.3 3.24	26.6 2.94	30.5 2.59	33.7 2.38	36.4 2.23
≤0.100	18.1 3.79	20.4 3.38	21.4 3.24	22.5 3.14	24.9 2.91	27.1 2.75	30.8 2.50	34.0 2.32	36.7 2.18
≤0.125	19.9 3.18	21.9 2.99	22.8 2.94	23.7 2.87	25.9 2.74	27.9 2.62	31.4 2.42	34.4 2.26	37.0 2.13
≤0.150	21.6 2.88	23.3 2.79	24.2 2.78	25.0 2.72	26.9 2.60	28.8 2.52	32.1 2.36	34.9 2.21	37.3 2.08
≤0.175	23.2 2.73	24.7 2.66	25.5 2.65	26.2 2.60	28.0 2.52	29.7 2.44	32.7 2.28	35.2 2.14	36.8 1.96
≤0.200	24.7 2.63	26.1 2.59	26.7 2.52	27.4 2.51	29.0 2.43	30.6 2.37	32.8 2.14	34.5 1.94	36.1 1.79
≤0.225	26.1 2.53	27.3 2.45	27.9 2.42	28.5 2.40	30.0 2.34	30.8 2.14	32.3 1.86	34.0 1.73	35.7 1.64
≤0.250	27.5 2.39	28.6 2.39	29.1 2.33	29.7 2.33	30.6 2.12	31.3 1.93	32.8 1.70	34.3 1.58	35.8 1.50

Source: AASHTO (2005)

Shear

Table 5.8.3.4.2-2 Values of θ and β for Sections with Less than Minimum Transverse Reinforcement.

s_{xe} (mm)	$\varepsilon_x \times 1000$										
	≤−0.20	≤−0.10	≤−0.05	≤0	≤0.125	≤0.25	≤0.50	≤0.75	≤1.00	≤1.50	≤2.00
≤130	25.4 6.36	25.5 6.06	25.9 5.56	26.4 5.15	27.7 4.41	28.9 3.91	30.9 3.26	32.4 2.86	33.7 2.58	35.6 2.21	37.2 1.96
≤250	27.6 5.78	27.6 5.78	28.3 5.38	29.3 4.89	31.6 4.05	33.5 3.52	36.3 2.88	38.4 2.50	40.1 2.23	42.7 1.88	44.7 1.65
≤380	29.5 5.34	29.5 5.34	29.7 5.27	31.1 4.73	34.1 3.82	36.5 3.28	39.9 2.64	42.4 2.26	44.4 2.01	47.4 1.68	49.7 1.46
≤500	31.2 4.99	31.2 4.99	31.2 4.99	32.3 4.61	36.0 3.65	38.8 3.09	42.7 2.46	45.5 2.09	47.6 1.85	50.9 1.52	53.4 1.31
≤750	34.1 4.46	34.1 4.46	34.1 4.46	34.2 4.43	38.9 3.39	42.3 2.82	46.9 2.19	50.1 1.84	52.6 1.60	56.3 1.30	59.0 1.10
≤1000	36.6 4.06	36.6 4.06	36.6 4.06	36.6 4.06	41.2 3.20	45.0 2.62	50.2 2.00	53.7 1.66	56.3 1.43	60.2 1.14	63.0 0.95
≤1500	40.8 3.50	40.8 3.50	40.8 3.50	40.8 3.50	44.5 2.92	49.2 2.32	55.1 1.72	58.9 1.40	61.8 1.18	65.8 0.92	68.6 0.75
≤2000	44.3 3.10	44.3 3.10	44.3 3.10	44.33 3.10	47.1 2.71	52.3 2.11	58.7 1.52	62.8 1.21	65.7 1.01	69.7 0.76	72.4 0.62

Source: AASHTO (2005)

Minimum Transverse Reinforcement

- We need some transverse reinforcement when the ultimate shear force is greater than ½ of shear strength from concrete and prestressing force

$$V_u > \varphi 0.5(V_c + V_p)$$

- If we need it, the minimum amount shall be

$$A_v \ge 0.083\sqrt{f'_c}\,\frac{b_v s}{f_y}$$

- Maximum Spacing
  - For $v_u < 0.125 f'_c$
    $s_{max} = 0.8\, d_v \le 600$ mm
  - For $v_u > 0.125 f'_c$
    $s_{max} = 0.4 d_v \le 300$ mm